Photonics: Project 1B (2021)

Pulse Propagation and Bit Rate in SMFs

« Clearly, the performance of a lightwave system can be improved considerably by operating near the zero-dispersion wavelength of the fiber and using optical sources with a relatively narrow spectral width. »

Govind P. Agrawal (1951 –) (New York: *Fiber-Optic Communication Systems*, Fourth Edition, Wiley, 2010, page 52)

The main goal of this project is to calculate the dispersion-induced limitations on fiber-optic communication systems. We are exclusively interested in long-haul links, namely when optical sources with a small spectral width and single-mode fibers are used. Also, we are mainly focused on estimating the maximum theoretical value for the bit rate of a single channel, i.e., we do not address herein multichannel systems (WDM or wavelength-division multiplexing). In fact, if we denote by B_0 the (maximum) channel bit rate and by Δf_{ch} the channel spacing in frequency units, then the so-called spectral efficiency of a WDM system is

$$\eta_s = \frac{B_0}{\Delta f_{\text{ch}}}.$$

One should note that, for coherent detection, it is possible to have $\eta_s > 1\,\text{b}\cdot\text{s}^{-1}\cdot\text{Hz}^{-1}$.

To estimate the bit rate in this project we will always consider, at the input, chirped Gaussian pulses of the form

$$\mathrm{A}(0,t) = \mathrm{A}_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^2 \right]$$

where C is the (dimensionless) chirp parameter and $\mathrm{A}_0 > 0$. Students are referred to the following **PowerPoint** file: ***Pulse Propagation in Single-Mode Fibers***.

To calculate the bit rate we will use the following expression

$$\boxed{\frac{\sigma^2}{\sigma_0^2} = \left(1 + C\,\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + \left(1+C^2\right)^2 \left(\frac{\beta_3 z}{4\sqrt{2}\,\sigma_0^3}\right)^2}$$

where σ is the RMS (root mean square) width of the pulse $\mathrm{A}(z,t)$ at point z. The longitudinal wavenumber (also known as the propagation constant) $\beta = \beta(\omega)$ is expanded in a Taylor series around the carrier frequency ω_0, such that

$$\boxed{\Omega = \omega - \omega_0} \quad \Rightarrow \quad \boxed{\beta(\omega) = \beta(\omega_0 + \Omega) = \sum_{m=0}^{\infty} \frac{\beta_m}{m!}\,\Omega^m},$$

where

$$\beta_m = \left(\frac{d^m \beta}{d\,\omega^m}\right)_{\omega=\omega_0}.$$

However, in this expansion, we only retain terms up to the third order. In this project we always consider that $\beta_2 \neq 0$ and $\beta_3 \neq 0$. One should note that

$$\boxed{\begin{gathered}\beta_0 = \beta(\omega_0) \\ \beta_1 = \left(\frac{d\beta}{d\omega}\right)_{\omega=\omega_0} = \frac{1}{v_g(\omega_0)} \\ \beta_2 = \left(\frac{d^2\beta}{d\omega^2}\right)_{\omega=\omega_0} = -\frac{1}{v_g^2(\omega_0)}\left(\frac{d v_g}{d\omega}\right)_{\omega=\omega_0} \\ \beta_3 = \left(\frac{d^3\beta}{d\omega^3}\right)_{\omega=\omega_0} = \frac{2}{v_g^3(\omega_0)}\left(\frac{d v_g}{d\omega}\right)^2_{\omega=\omega_0} - \frac{1}{v_g^2(\omega_0)}\left(\frac{d^2 v_g}{d\omega^2}\right)_{\omega=\omega_0}\end{gathered}},$$

where the group velocity is given by

$$\boxed{v_g(\omega) = \frac{1}{\dfrac{d\beta}{d\omega}} = \frac{1}{\beta'(\omega)}}.$$

Coefficient β_2 is called the GVD (group-velocity dispersion) whereas β_3 accounts for higher--order dispersion. For a given point z $(0 \leq z \leq L)$ along the fiber, the group delay is then

$$\boxed{\tau_g(z) = \frac{z}{v_g(\omega_0)} = \beta_1 z}.$$

Only when $\beta_3 = 0$ can we get a Gaussian pulse propagating along the fiber from the input (at $z = 0$) to the output (at $z = L$). Otherwise, if $\beta_3 \neq 0$, the pulse is not Gaussian along the fiber; it is only Gaussian at the input.

To simplify our equations, we introduce a normalized distance

$$\boxed{\xi = \frac{z}{L}} \quad \Rightarrow \quad \left[\!\left[\begin{array}{lll} \text{at the input} & \mapsto & \xi = 0 \\ \text{at the output} & \mapsto & \xi = 1 \end{array} \right.\right.$$

and a normalized pulse width

$$\boxed{\chi = \frac{\sigma^2}{\tau_0^2}} \quad \Rightarrow \quad \left[\!\left[\begin{array}{lll} \text{at the input} & \mapsto & \chi_0 = \dfrac{\sigma_0^2}{\tau_0^2} \\ \text{at the output} & \mapsto & \chi_1 = \dfrac{\sigma_1^2}{\tau_0^2} \end{array} \right.\right.$$

where we have also introduced a characteristic time τ_0 such that

$$\boxed{\tau_0 = \sqrt{|\beta_2|\, L}}\,.$$

Accordingly, we can easily find that

$$\boxed{\chi = \chi(\xi) = \chi_0 + \operatorname{sgn}(\beta_2)\, C\,\xi + \left(\frac{p}{\chi_0}\right)\left(1 + a^2\,\frac{p}{2\,\chi_0}\right)\xi^2}\,.$$

In writing this last equation we have also defined the two following (dimensionless) coefficients

$$\boxed{\begin{aligned} p &= \frac{1+C^2}{4} \geq \frac{1}{4} \\ a &= \left| \frac{\beta_3}{\beta_2 \tau_0} \right| \end{aligned}}.$$

Also, we have the following ratio

$$\boxed{\mu = \mu(\xi) = \frac{\chi(\xi)}{\chi_0}} \quad \Rightarrow \quad \left[\!\left[\begin{array}{lll} \text{at the input} & \mapsto & \mu_0 \equiv 1, \\ \text{at the output} & \mapsto & \mu_1 = \dfrac{\chi_1}{\chi_0}. \end{array} \right.\right.$$

However, we can readily see that the output pulse width depends on the input pulse width, i.e., we have $\chi_1 = \chi_1(\chi_0)$. Therefore, we are interested in the optimization procedure corresponding to calculate the minimum value of this function. Thereby, we calculate χ_0^{opt} and χ_1^{opt}.

One should note that, after this optimization procedure, the evolution (along the fiber) of $\mu(\xi)$ depends on the chirp parameter C. Therefore, we define a critical value of this parameter, $C = C_{\text{cr}}$, as the one that leads to

$$\boxed{C = C_{\text{cr}}} \quad \Rightarrow \quad \boxed{\mu_1^{\text{opt}} = 1}.$$

Then, we can calculate the bit rate as follows. Denoting by T_b the allocated bit slot, the bit rate is – by definition – the value

$$\boxed{B = \frac{1}{T_b}}.$$

Let us define, as a thumb rule,

$$\sigma(\xi) \le \sigma_{\max} = \frac{T_b}{4} = \frac{1}{4B}, \quad \underset{\xi}{\forall}\,(0 \le \xi \le 1).$$

This leads to a maximum value B_0 of the bit rate given by

$$\boxed{B_0 = \frac{1}{4\sigma_{\max}} = \frac{1}{4\,\tau_0\,\sqrt{\chi_{\max}}}}\,.$$

In your project you must present a logical sequence of reasons leading to the calculation of the bit rate along the lines previously stated. To illustrate that argument, you should present several plots. These plots can be readily obtained using the MATLAB scripts:

a_coefficient.m
cubic_equation.m
chi_1_vs_chi_0.m
chirp_critical_graphical.m
chirp_critical_solution.m
pulse_width_evolution.m
mu_vs_chirp.m
bit_rate.m
bit_rate_vs_length.m

Remark – *You should use the numerical values already inserted in those scripts. However, in writing your report, we should state all those numerical values that gave rise to those specific figures. Please note that these figures (although with different numerical values) were used in the aforementioned* **PowerPoint** *file.*